LECTURES ABOUT (ADVANCED) STATISTICAL PHYSICS

T.S.Biró, *MTA Wigner Research Centre for Physics, Budapest*

Lectures given at: University of Johannesburg, South-Africa,

November 26 – November 29, 2012.

1. **Ancient Thermodynamics (… - 1870)**
2. **The Rise of Statistical Physics (1890 – 1920)**
3. **Modern (postwar) Problems (1940 – 1980)**
4. **Corrections (1950 – 2005)**
5. **Generalizations (1960 – 2010)**
6. **High Energy Physics (1950 – 2010)**

LECTURE FOUR ABOUT (ADVANCED) STATISTICAL PHYSICS

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Generalized kinetic theory

Simulation using non-additive rule

PRL 95: 162302, 2005 **with Gábor Purcsel**

Non-extensive Boltzmann Equation (NEBE) with Gábor Purcsel:

- Rényi-Tsallis energy addition rule
- random momenta accordingly
- pairwise collisions repeated
- momentum distribution collected

Boltzmann algorithm: pairwise combination + separation

With additive composition rule at independence:

$$f_1 \cdot f_2 = f_3 \cdot f_4$$

$$E_1 + E_2 = E_3 + E_4$$

$$f_{\mathrm{eq}} = \frac{1}{Z} e^{-\beta E}$$

Such rules generate exponential distribution

Boltzmann algorithm: pairwise combination + separation

With associative composition rule at independence:

$$f_1 \cdot f_2 = f_3 \cdot f_4$$

$$E_1 \oplus E_2 = E_3 \oplus E_4$$

$$L(E_1) + L(E_2) = L(E_3) + L(E_4)$$

$$f_{eq} = \frac{1}{Z} e^{-\beta L(E)}$$

Such rules generate 'exponential of the formal logarithm' distribution

Generalized Stoßzahlansatz

$$DF(f_1) = \int_{234} w_{1234}\left(G_{34} - G_{12}\right)$$

$$DF = \frac{p^\mu}{p^0}\partial_\mu F$$

$$G_{ij} = e_a\left(\ln_a(f_i) + \ln_a(f_j)\right)$$

General H theorem

$$S^{\mu} = -\int \frac{p^{\mu}}{p^{0}} \sigma(F(f))$$

$$\partial_{\mu} S^{\mu} = -\int_{1} \sigma'(F) DF(f_{1}) =$$

$$\frac{1}{4} \int_{1234} w_{1234} (\sigma'_{1} + \sigma'_{2} - \sigma'_{3} - \sigma'_{4})(G_{12} - G_{34})$$

$$\partial_{\mu} S^{\mu} \geq 0 \quad \text{iff} \quad \Phi(\mathrm{G}_{\mathrm{ij}}) = \sigma'_{i} + \sigma'_{j}$$

a monotonic rising function

General H theorem: entropy density formula

$$\Phi(G) = \alpha \ln_a(G) + 2\beta$$

$$\sigma'(F(f)) = \alpha \ln_a(f) + \beta$$

$$\sigma(f) = \int F'(f)\left(\alpha \ln_a(f) + \beta\right)df$$

Detailed balance: $G_{12} = G_{34}$

Stoßzahlan satz :

$$G_{ij} = e_a\left(\ln_a f_i + \ln_a f_j\right) =$$

$$e^{L_a\left(L_a^{-1}(\ln f_i) + L_a^{-1}(\ln f_j)\right)}$$

Conserved in microcolli sion :

$$\mathrm{X}(\mathrm{E}_1) + \mathrm{X}(\mathrm{E}_2) = \mathrm{X}(\mathrm{E}_3) + \mathrm{X}(\mathrm{E}_4)$$

Detailed balance: $G_{12} = G_{34}$

General solution :

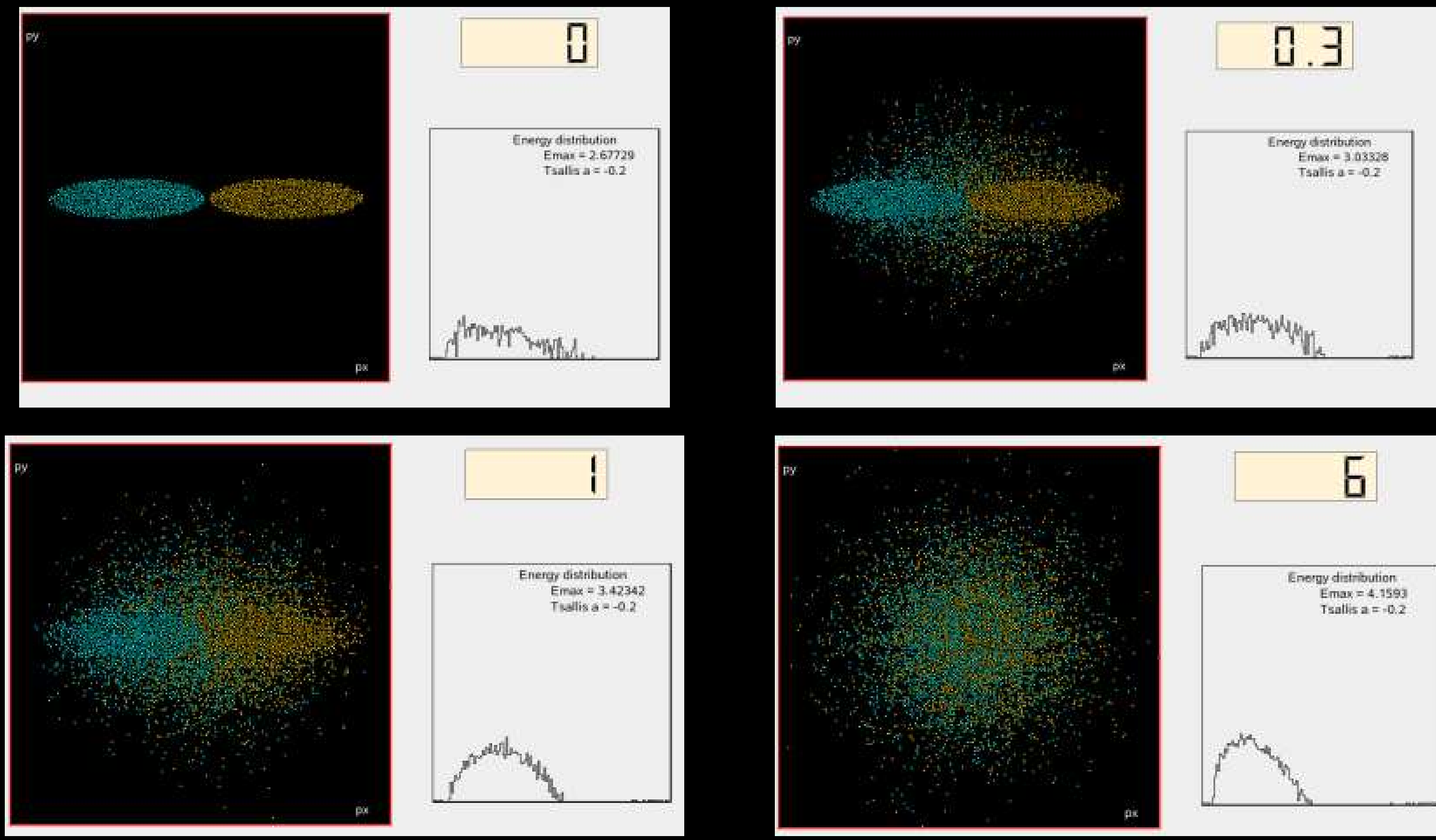
$$L_a^{-1}(\ln f_i) = \alpha - \beta X(E_i)$$

$$f_i = \exp\left(L_a\left[\alpha - \beta X(E_i)\right]\right)$$

Detailed balance: proof

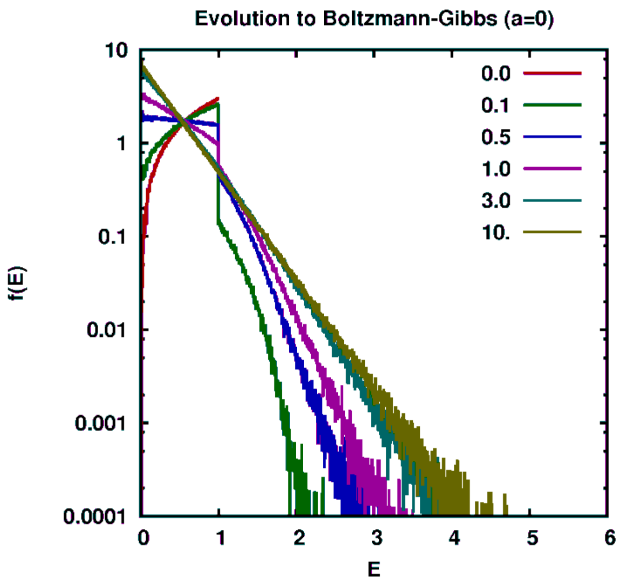
$$G_{12} = \exp\left(L_a\left[L_a^{-1}(\ln f_1) + L_a^{-1}(\ln f_2)\right]\right)$$
$$= \exp\left(L_a\left[2\alpha - \beta X(E_1) - \beta X(E_2)\right]\right)$$
$$= \exp\left(L_a\left[2\alpha - \beta X(E_3) - \beta X(E_4)\right]\right)$$
$$\exp\left(L_a\left[L_a^{-1}(\ln f_3) + L_a^{-1}(\ln f_4)\right]\right) = G_{34}$$

Evolution in NEBE phase space

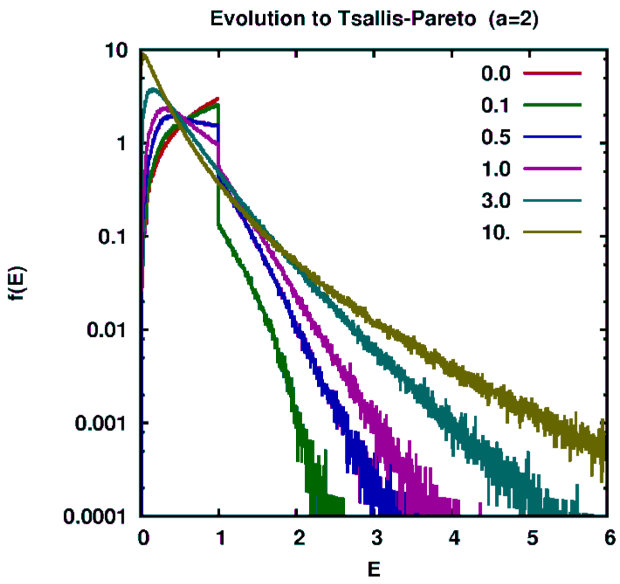


Stationary energy distributions in NEBE program

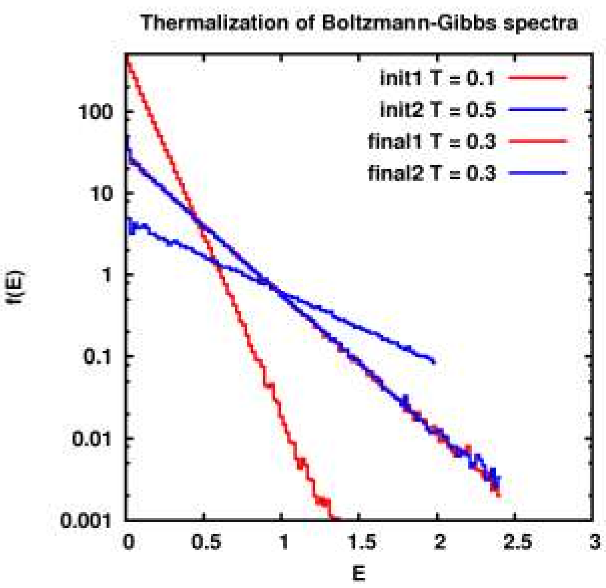
x + y

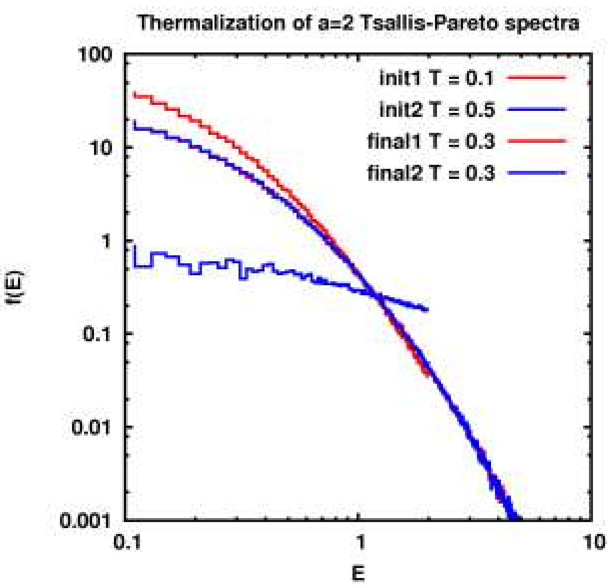


x + y + 2 x y



Thermal equilibration in NEBE program





Microscopic theory in non-extensive approach: questions, projects, ...

- Ideal gas with deformed exponentials
- Boltzmann and Bose distribution
- Fermi distribution: ptl – hole effect
- Thermal field theory with stohastic temperature (Euler-Gamma distribution)
- Lattice SU(2) with non-ext. Metropolis method

Ideal Tsallis-Bose gas

$$\ln Z = V \ldots \int E^2 dE \left(-\ln(1 - e_{-a}(-\beta E)) \right)$$

$$-\frac{1}{V}\frac{\partial \ln Z}{\partial \beta} = \ldots \int E^2 dE \frac{E}{1 + a\beta E} \frac{1}{e_a(\beta E) - 1}$$

$$e = \frac{\gamma T^4}{2\pi^2} \sum_{n=1}^{\infty} \frac{3!}{n(n-a)(n-2a)(n-3a)}$$

For c = 5.5 we have 1 / a = 4.5 and e ≈ 4 e_0

Limiting temperature with Tsallis distribution

PLB 632:247,2006 (**with A. Peshier, Giessen**)

Massless particles, 3-dim. momenta, N-fold

Hagedorn

$$3T = \frac{E/N}{1 + a\,E/N}$$

For N ≥ 4: Tsallis partons ➔ Hagedorn states

If it were a ~ 1/T → there would not be a limiting T !!!

Tsallis lattice EOS

Tamás S. Bíró (KFKI RMKI Budapest) and
Zsolt Schram (DTP ATOMKI Debrecen)

- **Lattice action with superstatistics**
- **Ideal gas with power-law tails**
- **Numerical results on EOS**

Lattice theory

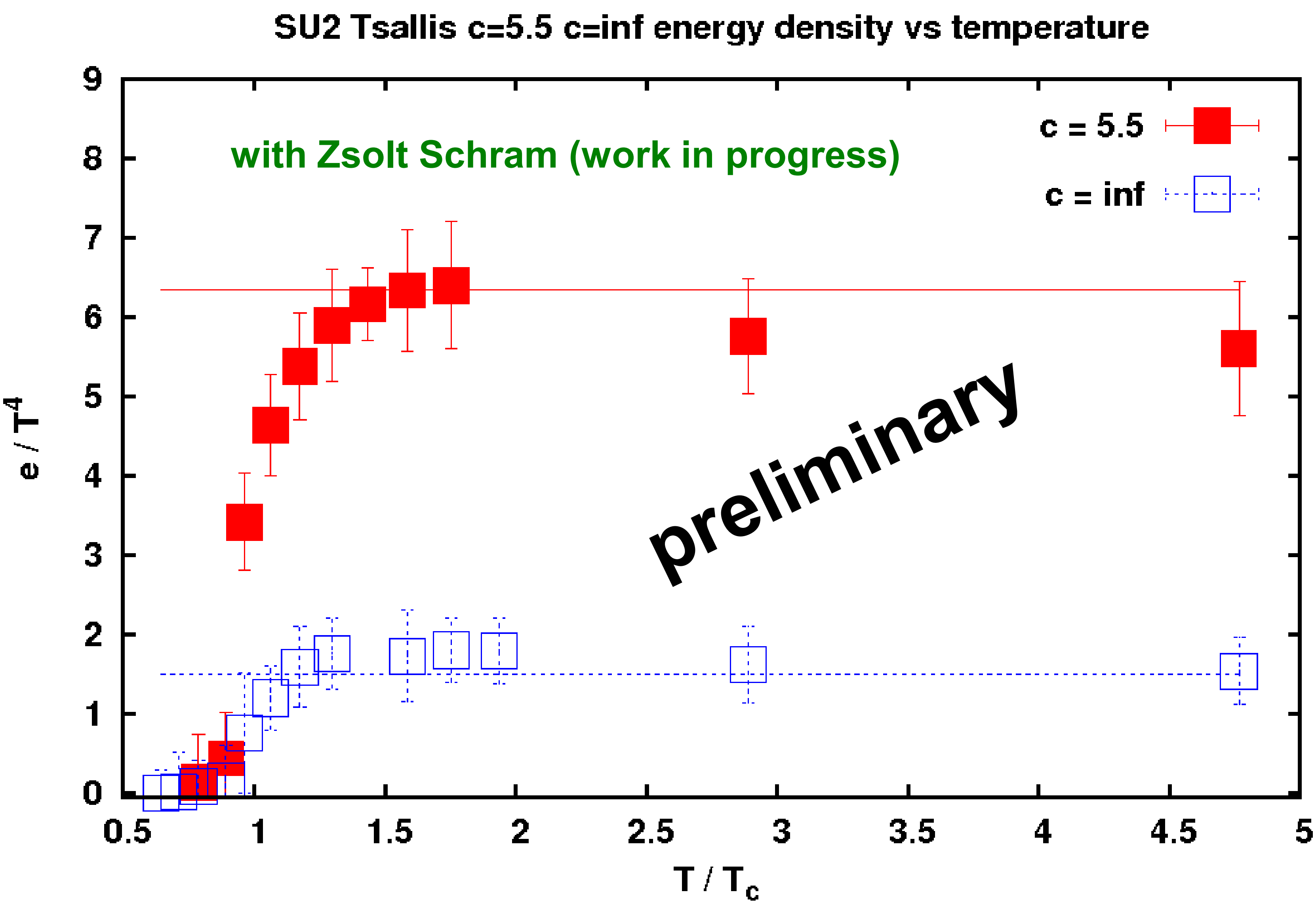
Expectation values of observables:

$$\langle A \rangle = \frac{\int DU \int dt\, w_c(t)\; e^{-S(t,U)}\; {}^{v}t\; A(U)}{\int DU \int dt\, w_c(t)\; e^{-S(t,U)}}$$

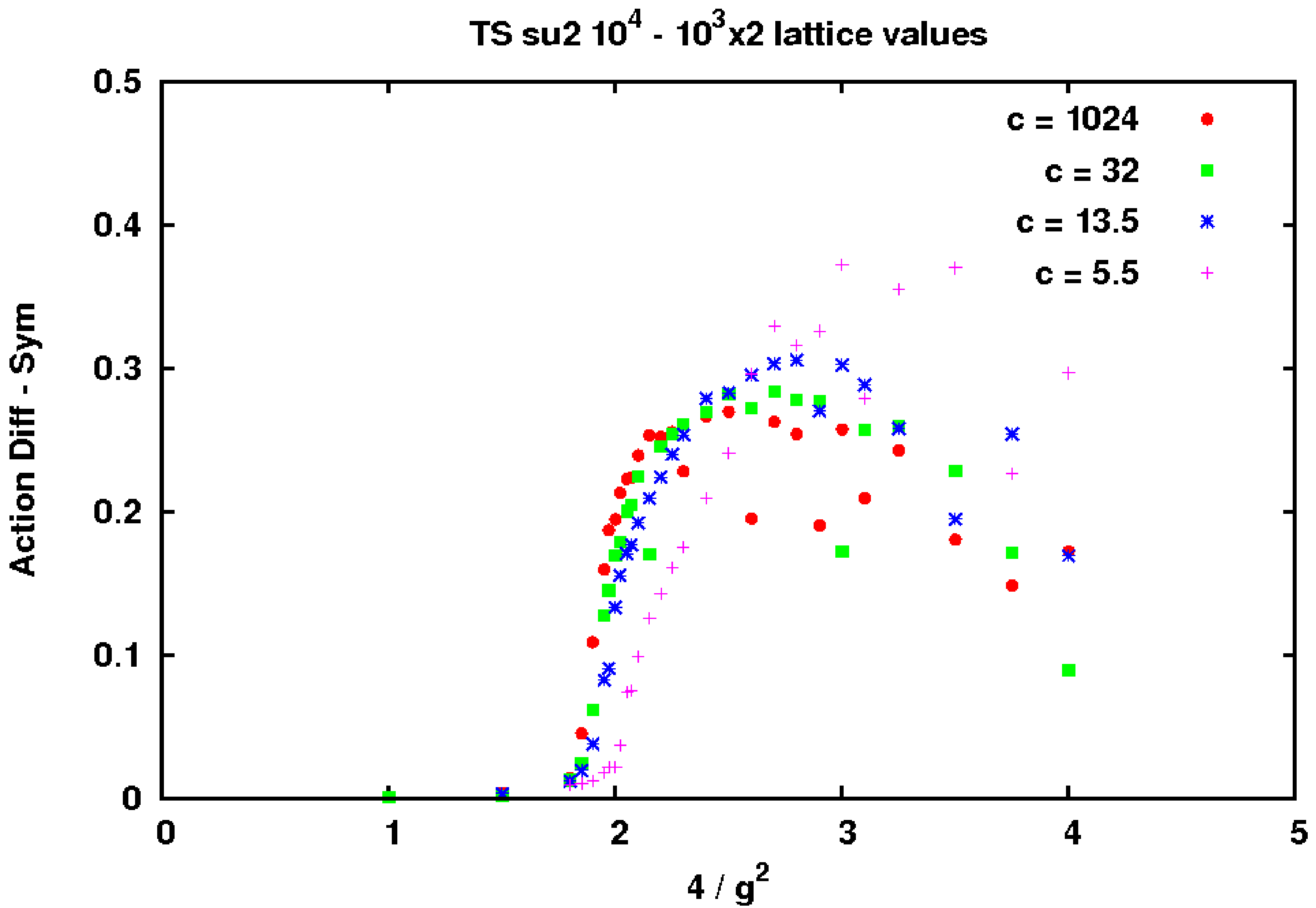
Action: $S(t,U) = a(U)\,t + b(U)\,/\,t$

$t = a_t \,/\, a_s$ *asymmetry parameter*

Su2 Yang-Mills eos on the lattice with Euler-Gamma distributed inverse temperature: Effective action method

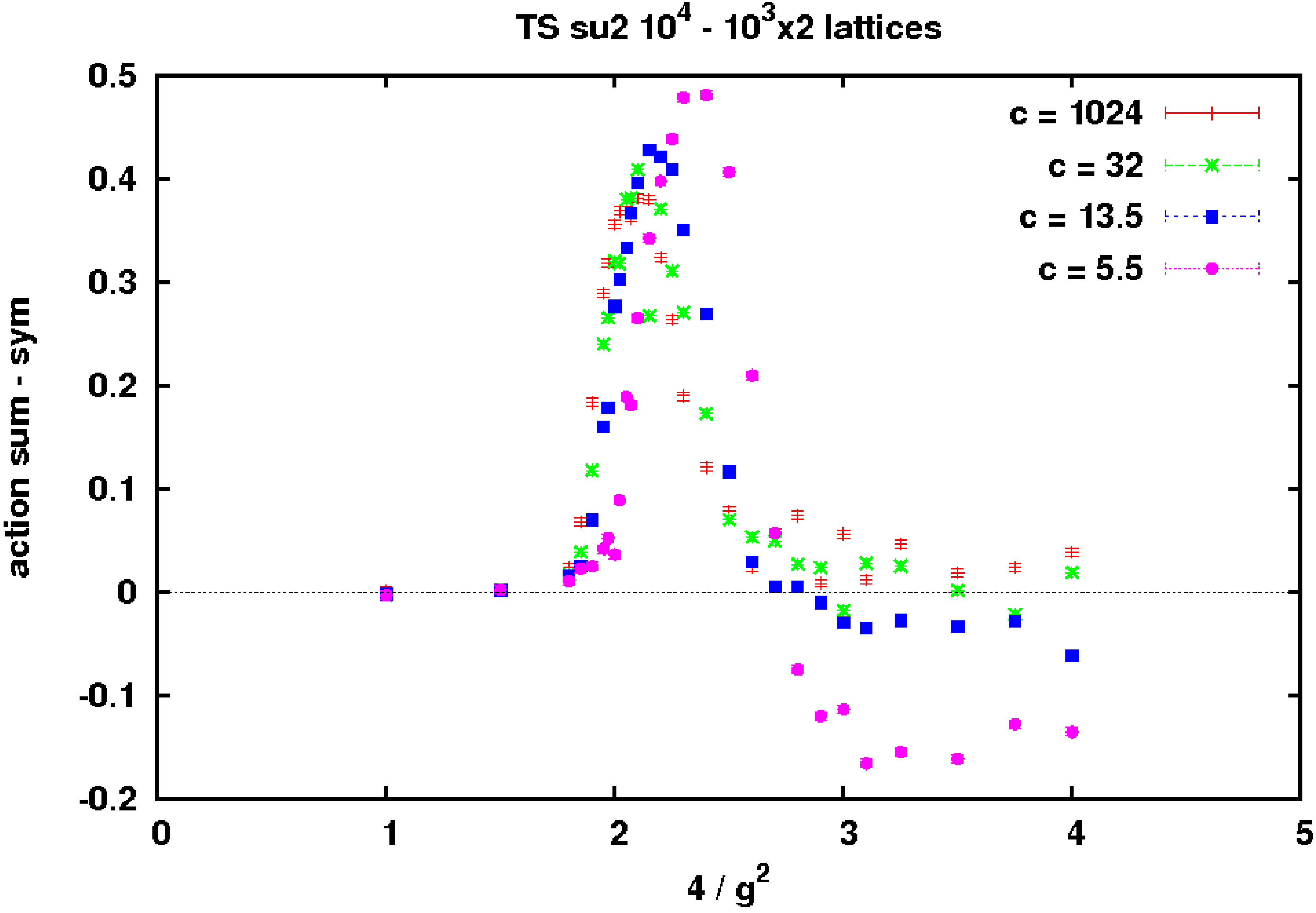


e / T4



TS su2 10^4 - 10^3x2 lattice values
Action Diff - Sym
4 / g^2
c = 1024
c = 32
c = 13.5
c = 5.5
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
5

(e-3p) / T4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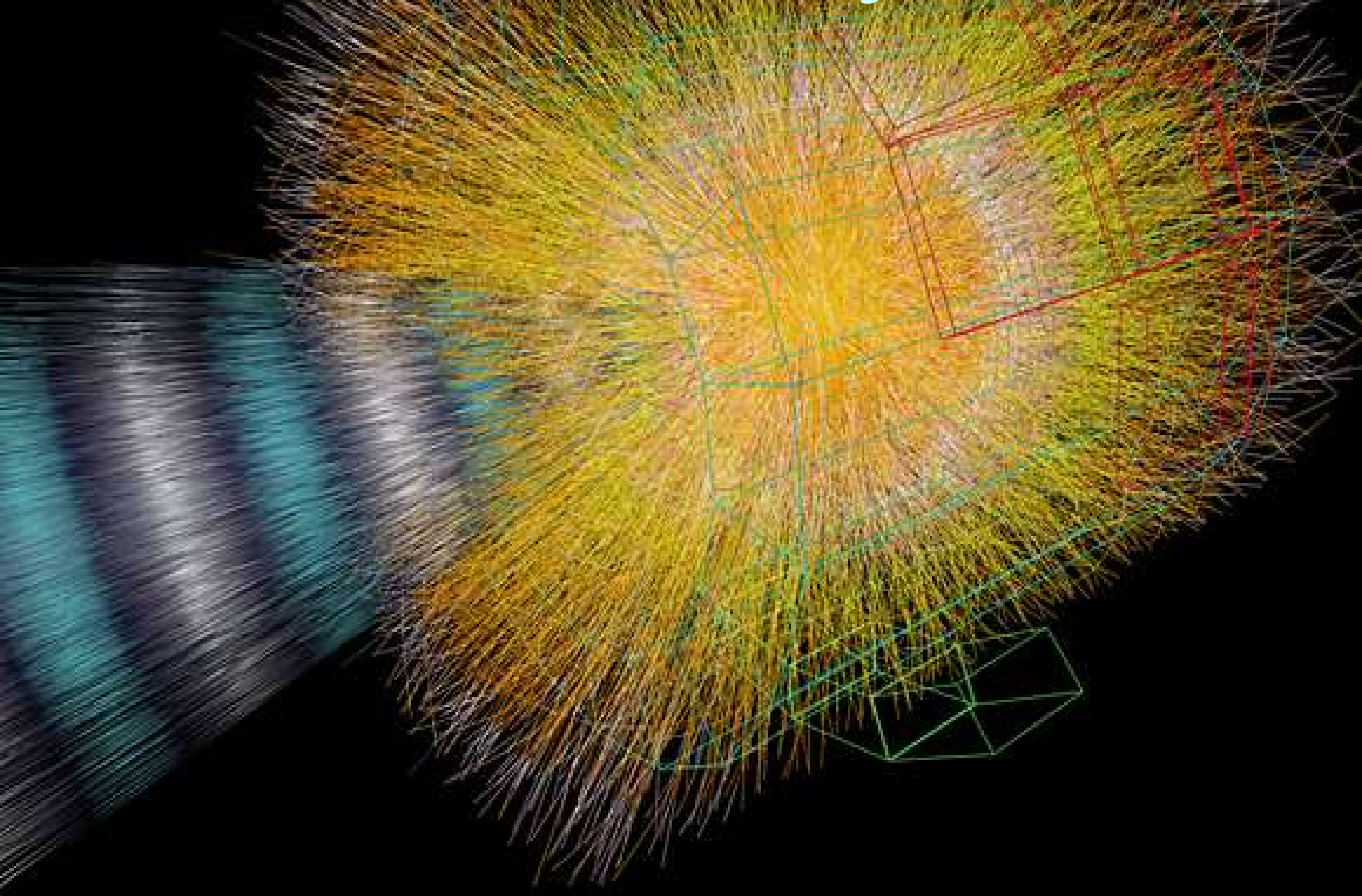
0.3
0.2
0.1
0
-0.1
-0.2
0
1
2
3
4
5

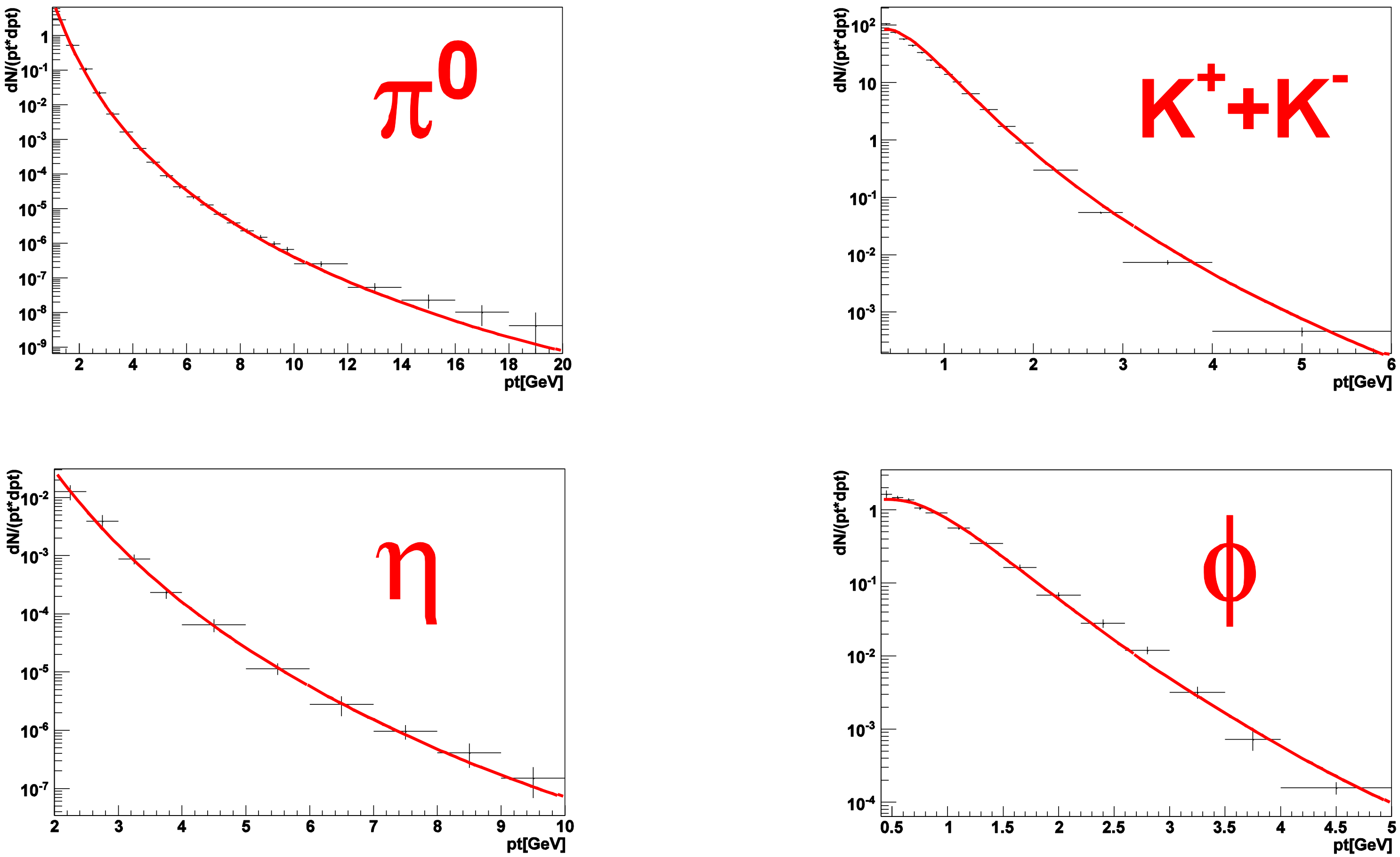
HIGH ENERGY PHYSICS

- **Old and New Statistical Models, Hydrodynamics**
- **Multiplicity Fluctuations**
- **Valence Quark Scaling**
- **Non-extensive Boltzmann Equation**
- **Fragmentation Statistics**
- **Classical Fields and Thermal Gravity**

RHIC: Relativistic Heavy Ion Collider

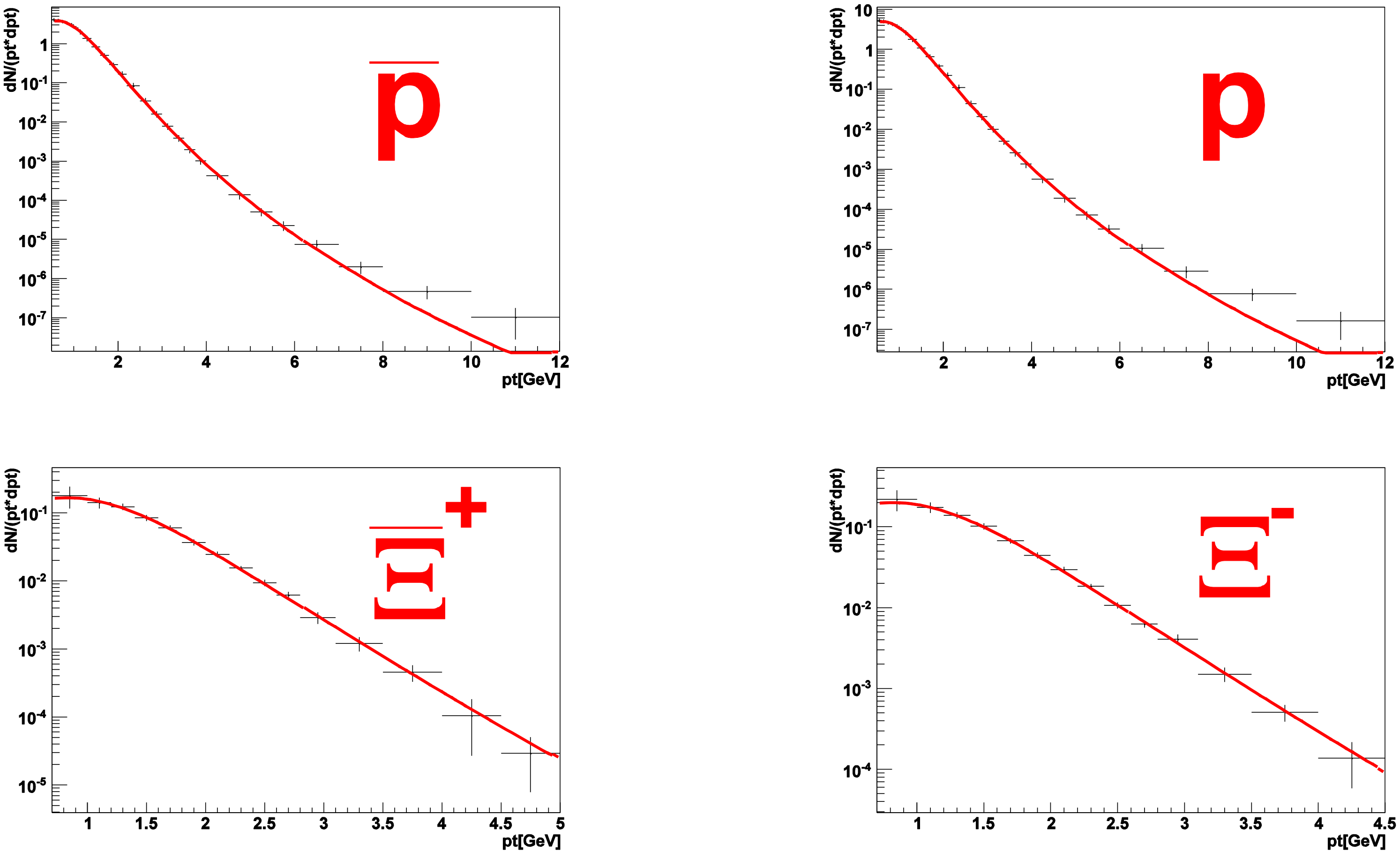


Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

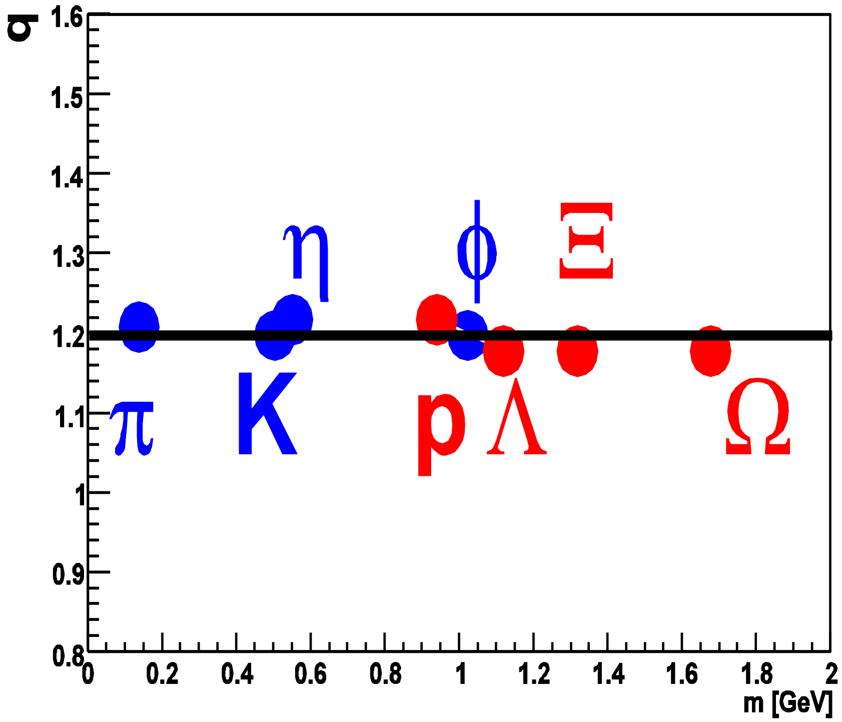


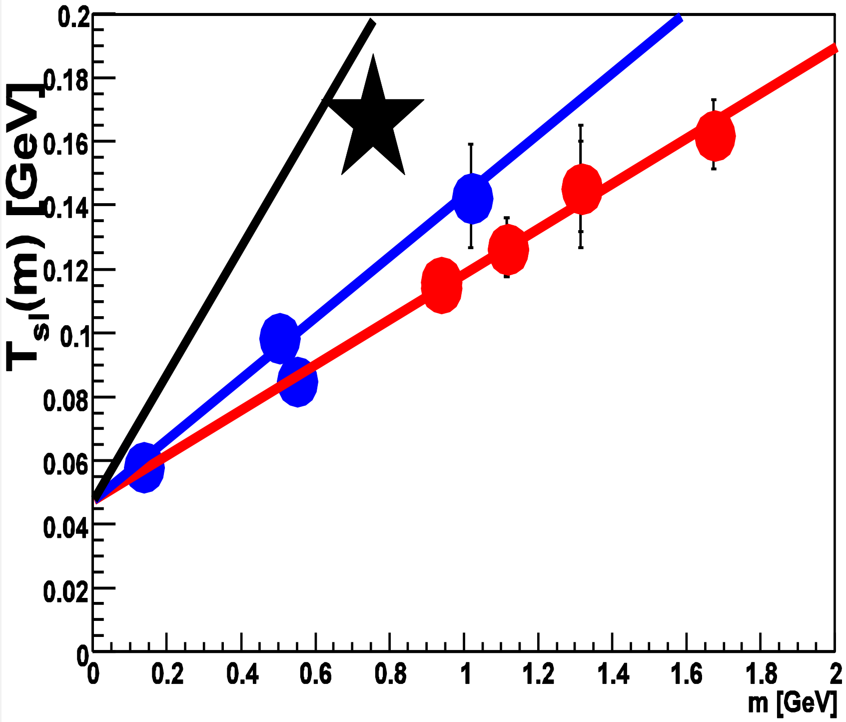
SQM 2008, Beijing

Tsallis quark matter + transverse flow + quark coalescence fits to hadron spectra

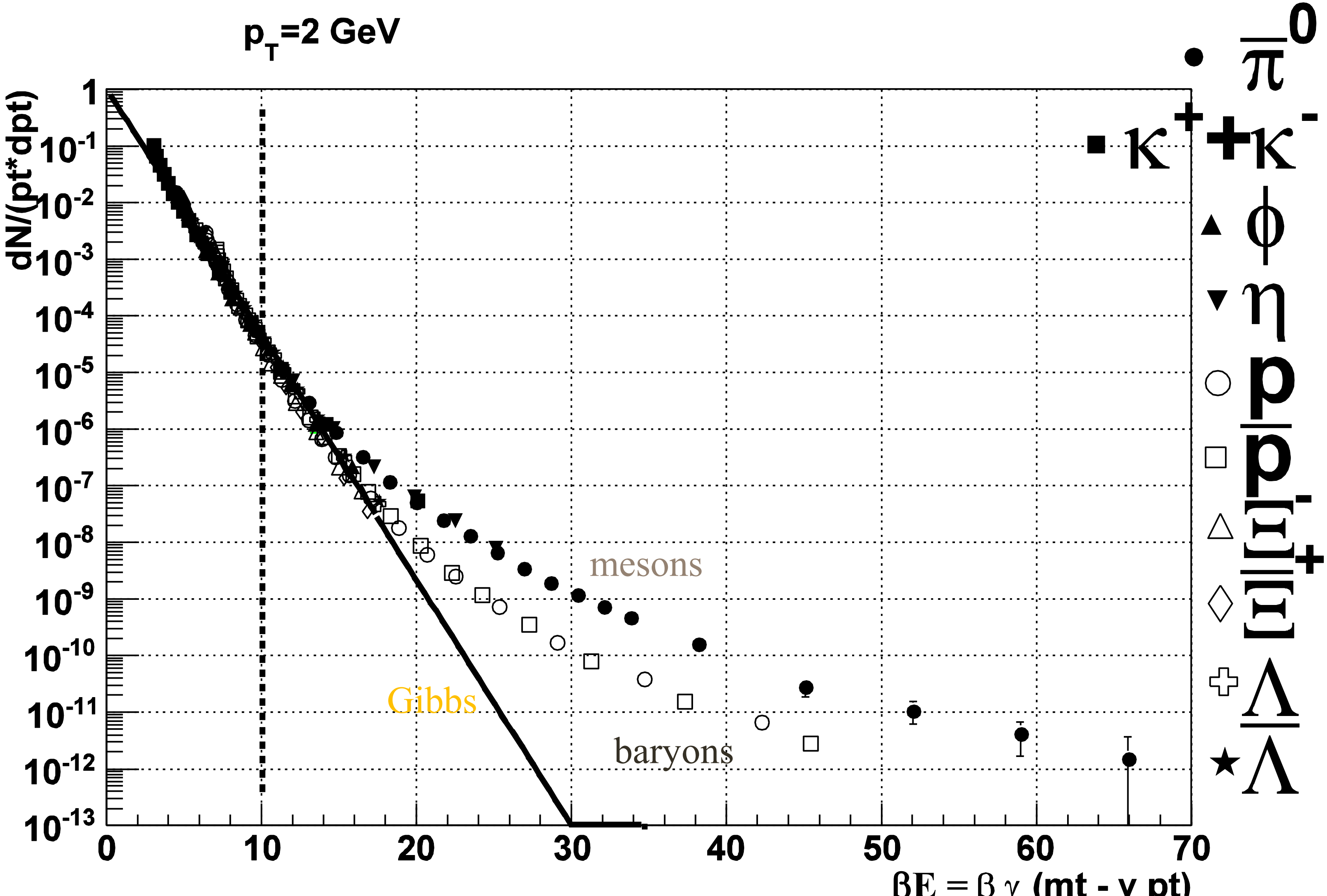


Blast wave fits and quark coalescence





WHAT DOES SCALE, E OR L(E)?



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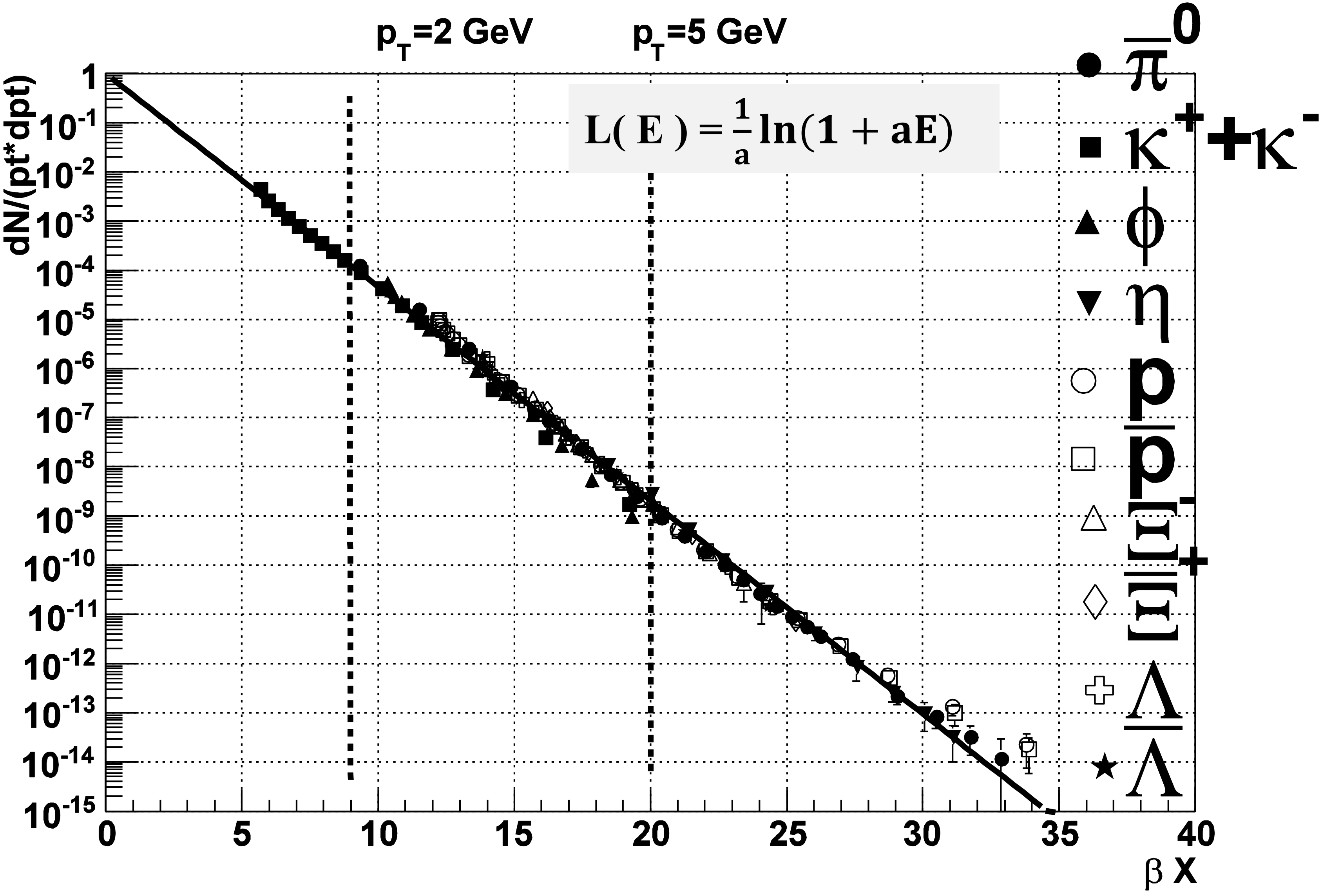

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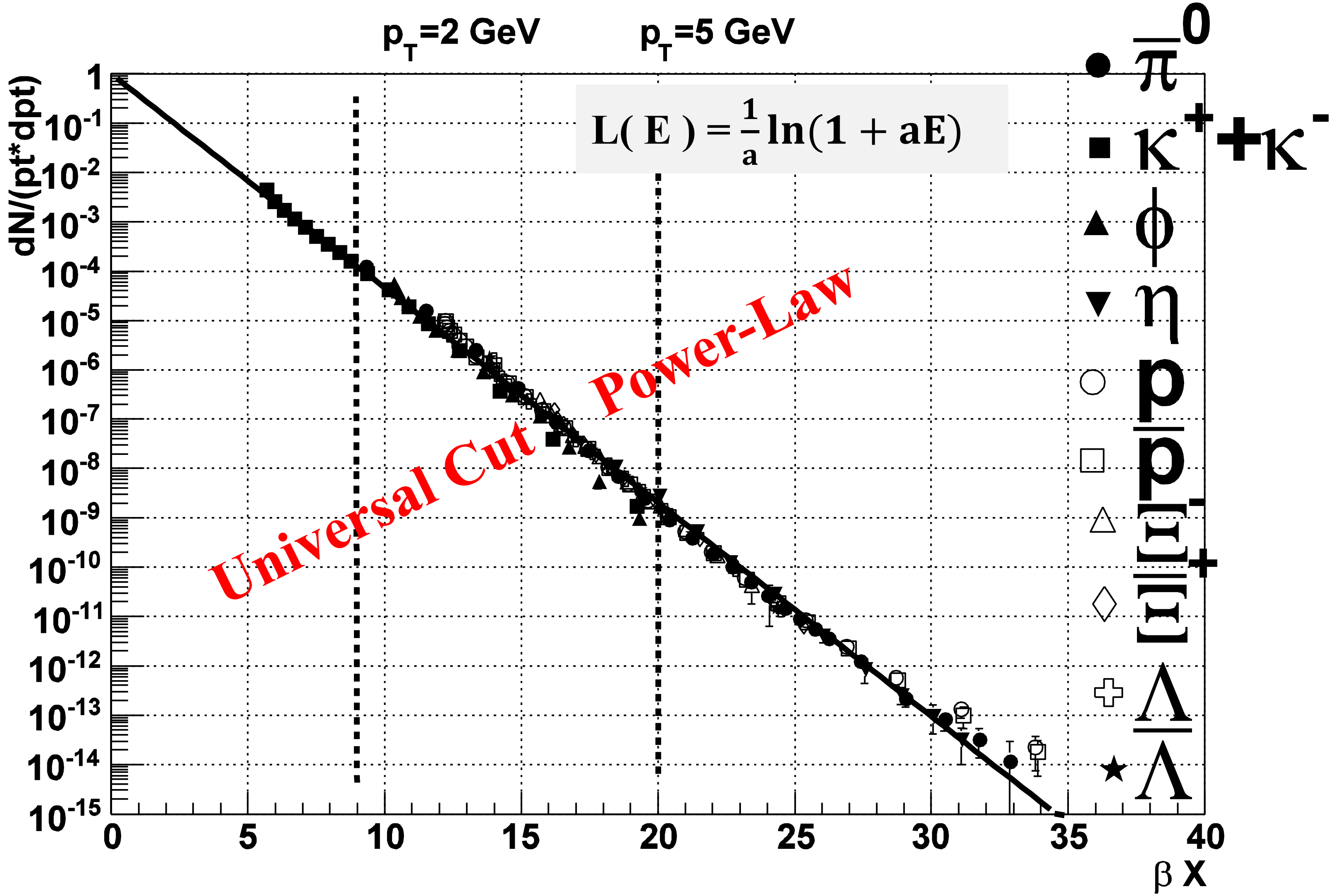


LIMITING TEMPERATURE WITH TSALLIS DISTRIBUTION

(with A. Peshier, Giessen)

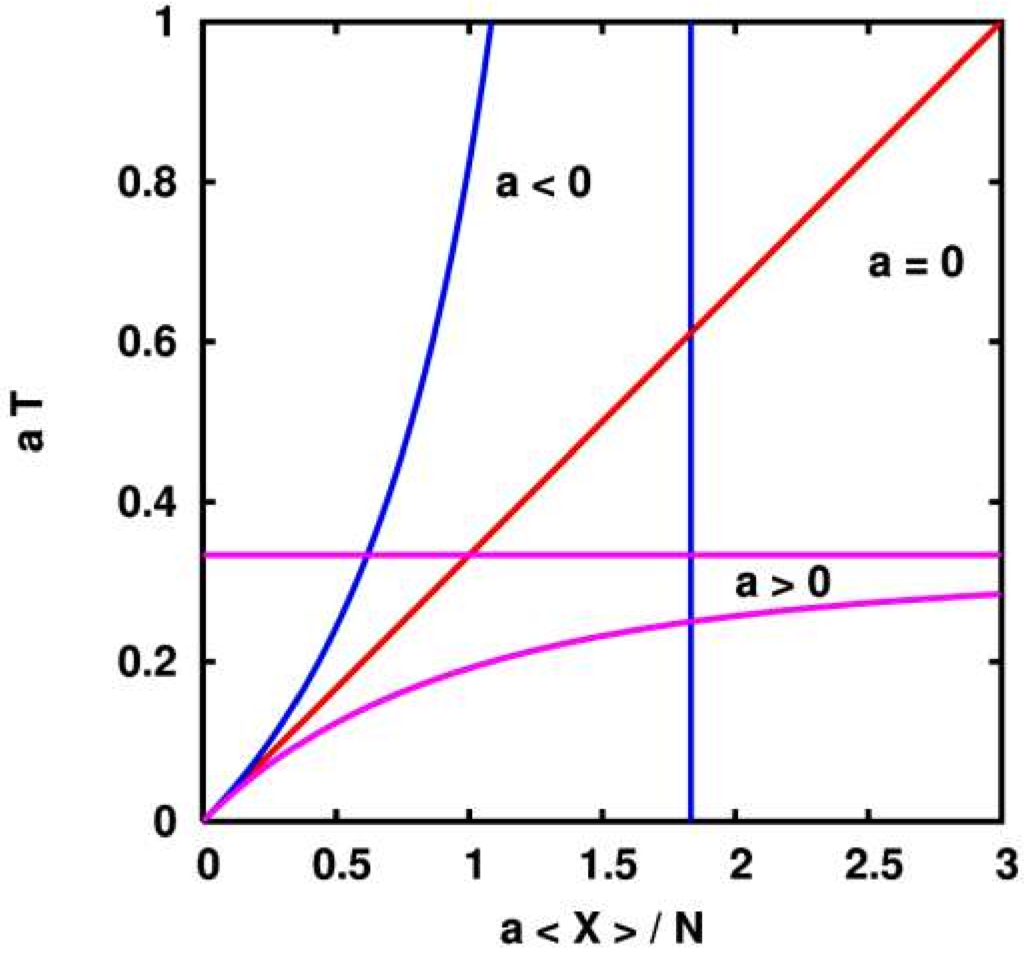
hep-ph/0506132, PLB

Massless particles, d-dim. momenta, N-fold

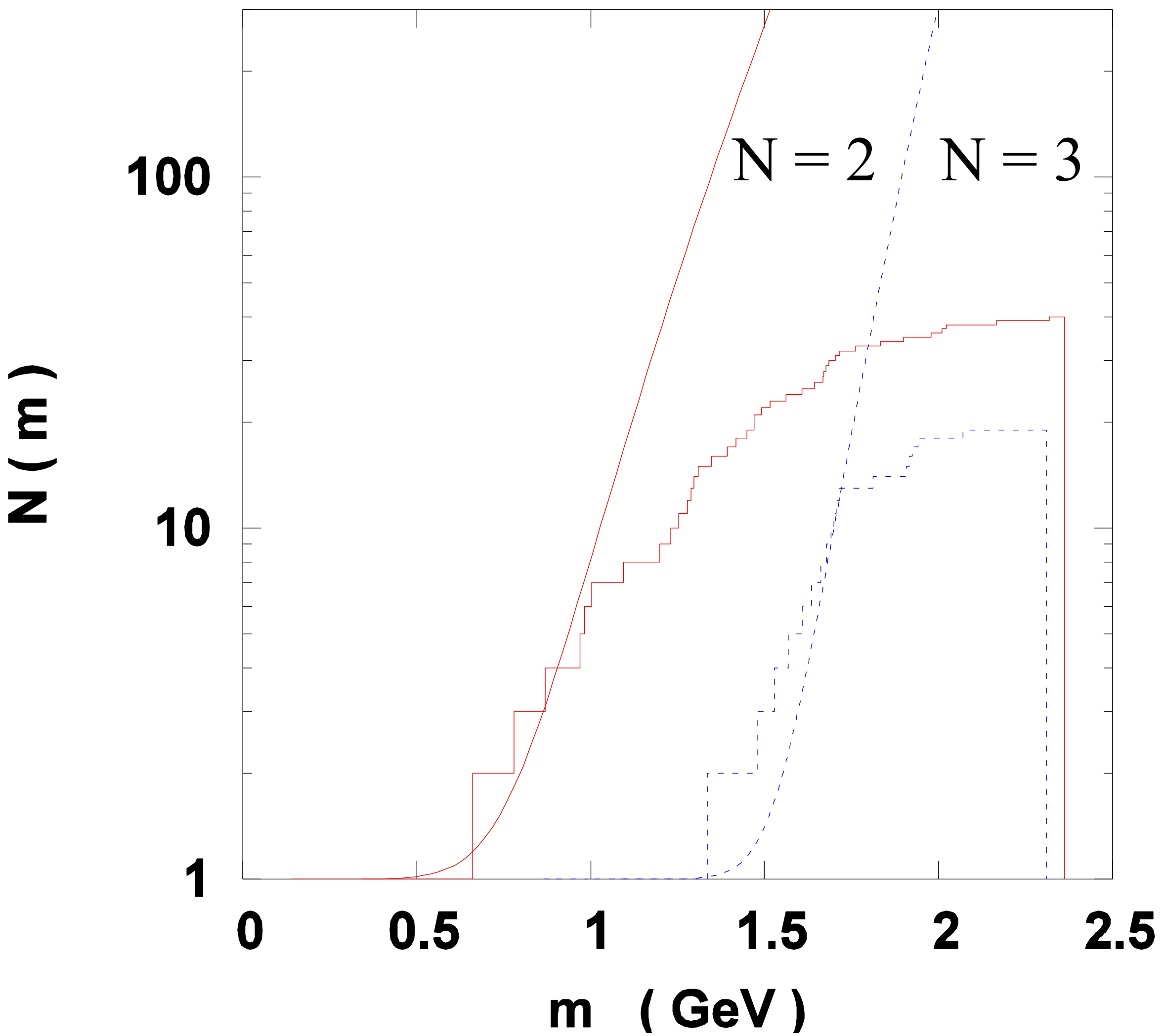
$$\frac{\langle X(E)\rangle}{N} = \sum_{j=1}^{d} \frac{T E_c}{E_c - j\,T} \; ; \qquad T_H = E_c \,/\, d$$

For $N \geq 2$: Tsallis partons → Hagedorn hadrons

TEMPERATURE VS. ENERGY



HADRON MASS SPECTRUM FROM X(E)-FOLDING OF TSALLIS



The hadronization line in stringy quark matter

T.S.Bíró, MTA KFKI RMKI Budapest, Hungary
J. Cleymans, University of Cape Town, South Africa

- How can be E / N = 6 T ?
- Stringy corrections to QGP
- High-T equation of state
- The zero pressure line

NFQCD Workshop, Yukawa Institute, Kyoto, February 13, 2008.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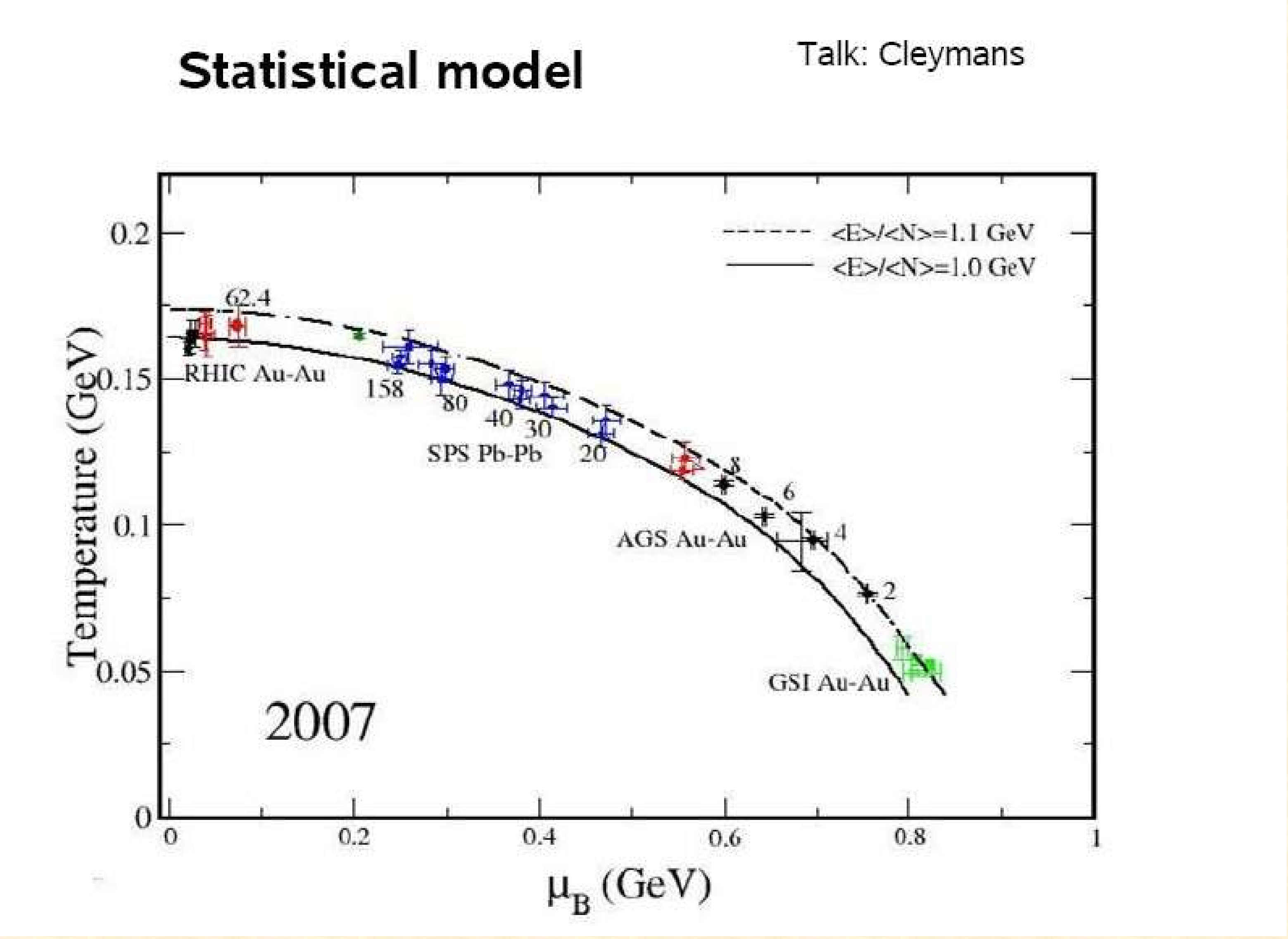
Statistical Model



How can be E / N = 6 T ?

$$T = 167\ MeV$$

$$E / N = 1\ GeV$$

Statistical Model: hadronization point around μ = 0 (RHIC, LHC)

How can be E / N = 6 T ?

$$E\,/\,N = m + \tfrac{3}{2}\,T$$

$$m = 750\ MeV$$

Massive hadrons (rho?)

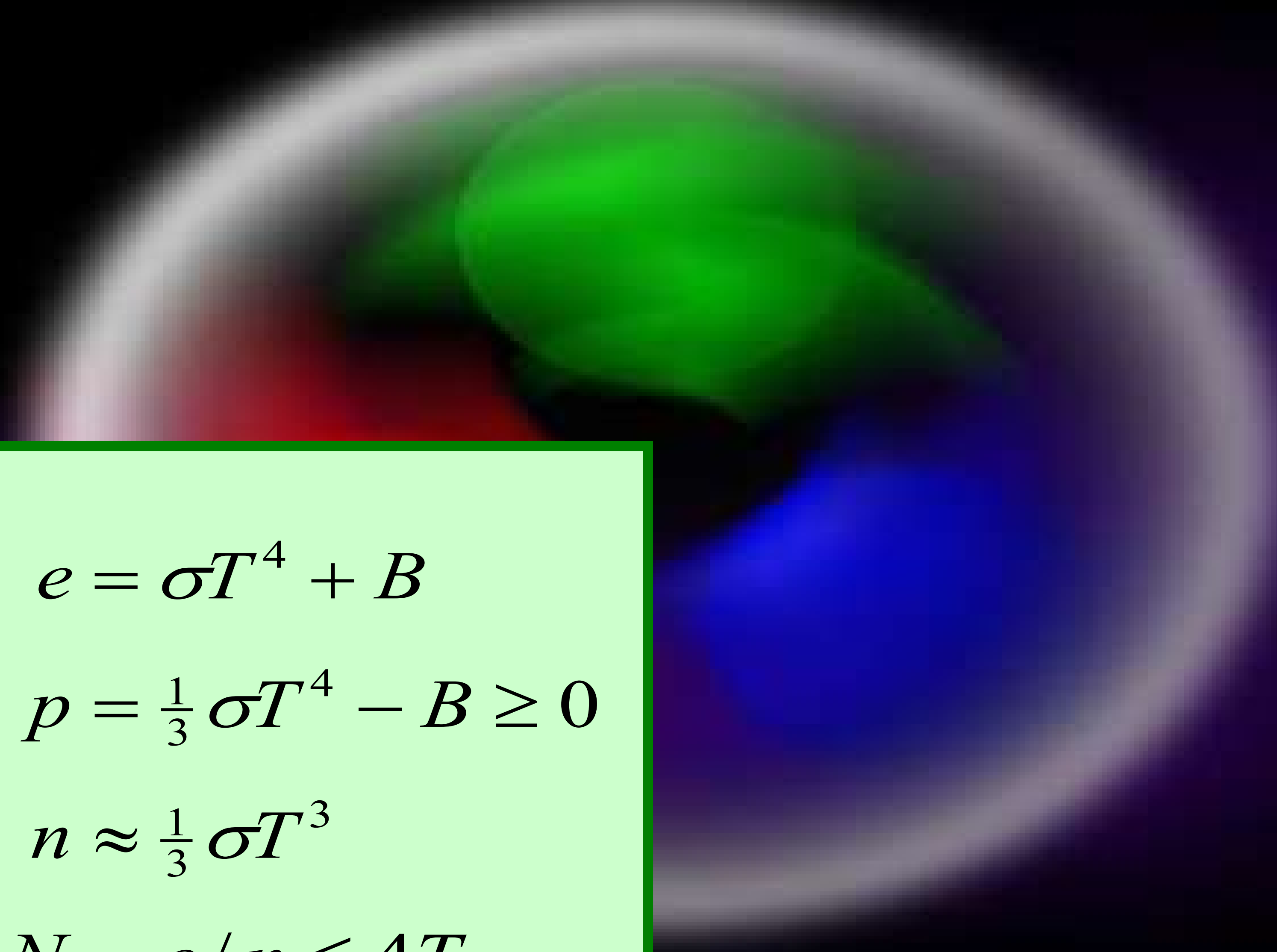
How can be E / N = 6 T ?

$$E / N = 3T$$

$$m = 0$$

Ideal gas of radiation

How can be E / N = 6 T ?



$$e = \sigma T^4 + B$$

$$p = \tfrac{1}{3}\sigma T^4 - B \geq 0$$

$$n \approx \tfrac{1}{3}\sigma T^3$$

$$E / N = e / n \leq 4T$$

Bag Model for QGP

How can be E / N = 6 T ?

Stringy Massless QGP

How can be E / N = 6 T ?

Stringy Massless QGP

$$n \cdot \sigma \langle \ell \rangle = \sigma \, g \, n^{-1/3} n = \tfrac{3}{2} A n^{2/3}$$

How can be E / N = 6 T ?

This is more generally true!

- **correction depends on**

$$c = \sum_i c_i \, n_i$$

- **constituents are massless**

$$p_{id} = T^4 \cdot \phi\left(\mu_B / T\right)$$

- **Boltzmann approximation is acceptable**

$$p_{id} = T \cdot \sum_i n_i$$

Stringy corrections to QGP

$$c = \sum_i c_i n_i$$

color density

$$f = f_{id} + \frac{1}{1-\gamma} A c^{1-\gamma}$$

free energy density
fractional power

$$\mu_i = \mu_{id} + A c^{-\gamma} c_i = q_i \mu_B$$

chemical potential

$$p = p_{id} - \frac{\gamma}{1-\gamma} A c^{1-\gamma}$$

pressure

$$e = e_{id} + \frac{1}{1-\gamma} A c^{1-\gamma}$$

energy density

Stringy corrections to QGP

Consistent solution for color density in equilibrium:

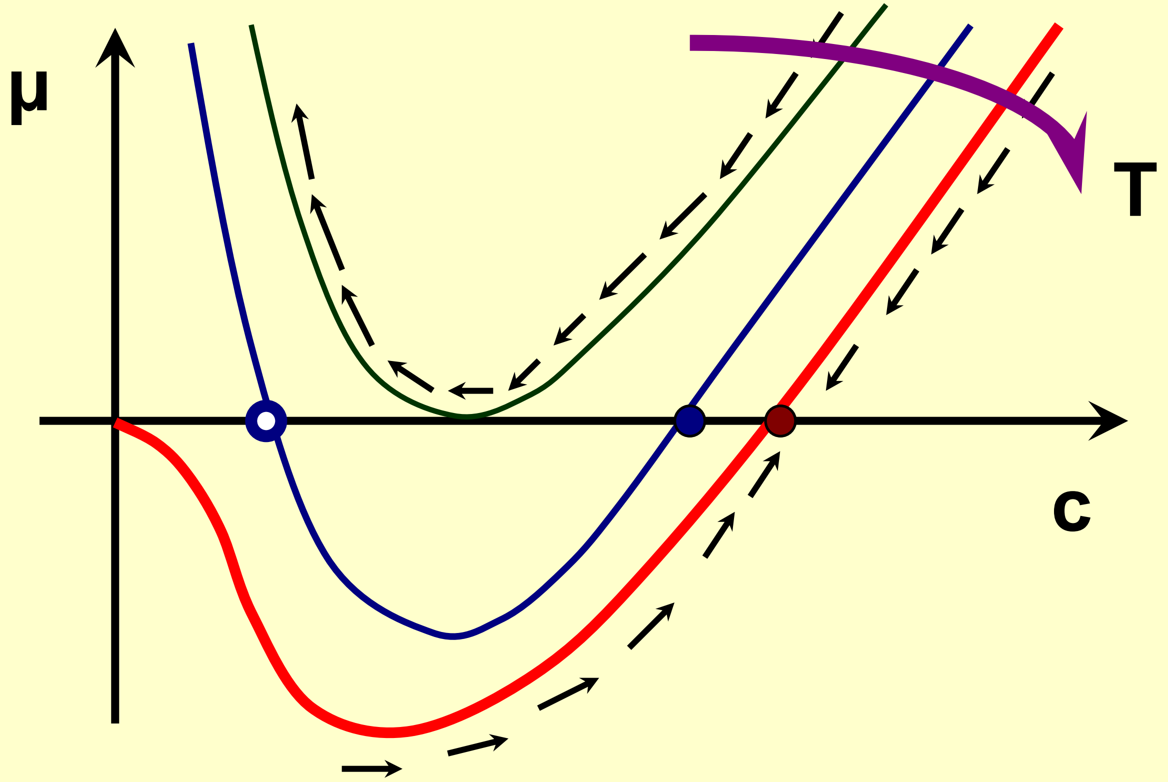
$$c = \sum_i c_i \, d_i \, n_{id}\left(T, q_i \mu_B - A c_i c^{-\gamma}\right)$$

$$\Rightarrow \quad c = c\,(T, \mu_B)$$

It exists only beyond an endline in T - μ

Stringy corrections to QGP

Consistent solution for color density in equilibrium:



It exists only beyond an endline in T - μ

Stringy corrections to QGP

One massless component, Boltzmann approximation:

$$c = \frac{d}{\pi^2} T^3 \, e^{q\mu_B/T} \, e^{-Ac^{-\gamma}/T}$$

$$z = \frac{A}{T} c^{-\gamma}, \quad \alpha = \frac{A}{T} \left(\frac{d}{\pi^2} T^3 \, e^{q\mu_B/T} \right)^{-\gamma}$$

$$z = \alpha \, e^{\gamma z}, \quad z = -\frac{1}{\gamma} W(-\gamma \alpha)$$

The solution is related to Lambert's function

Stringy corrections to QGP

Endline: last possible solution

Boltzmann approximation

$$z = \alpha\, e^{\gamma z}, \quad 1 = \gamma \alpha\, e^{\gamma z}$$

$$\Rightarrow \; z_E = \tfrac{1}{\gamma} \qquad \Rightarrow \qquad \alpha_E = \tfrac{1}{\gamma} e^{-1}$$

$$\mu_B = \frac{3\gamma + 1}{\gamma q}\, T \ln \frac{T_E}{T}$$

It is near to the zero pressure line for low chemical potential

Stringy corrections to QGP

End temperature: last possible solution at μ = 0

Boltzmann approximation

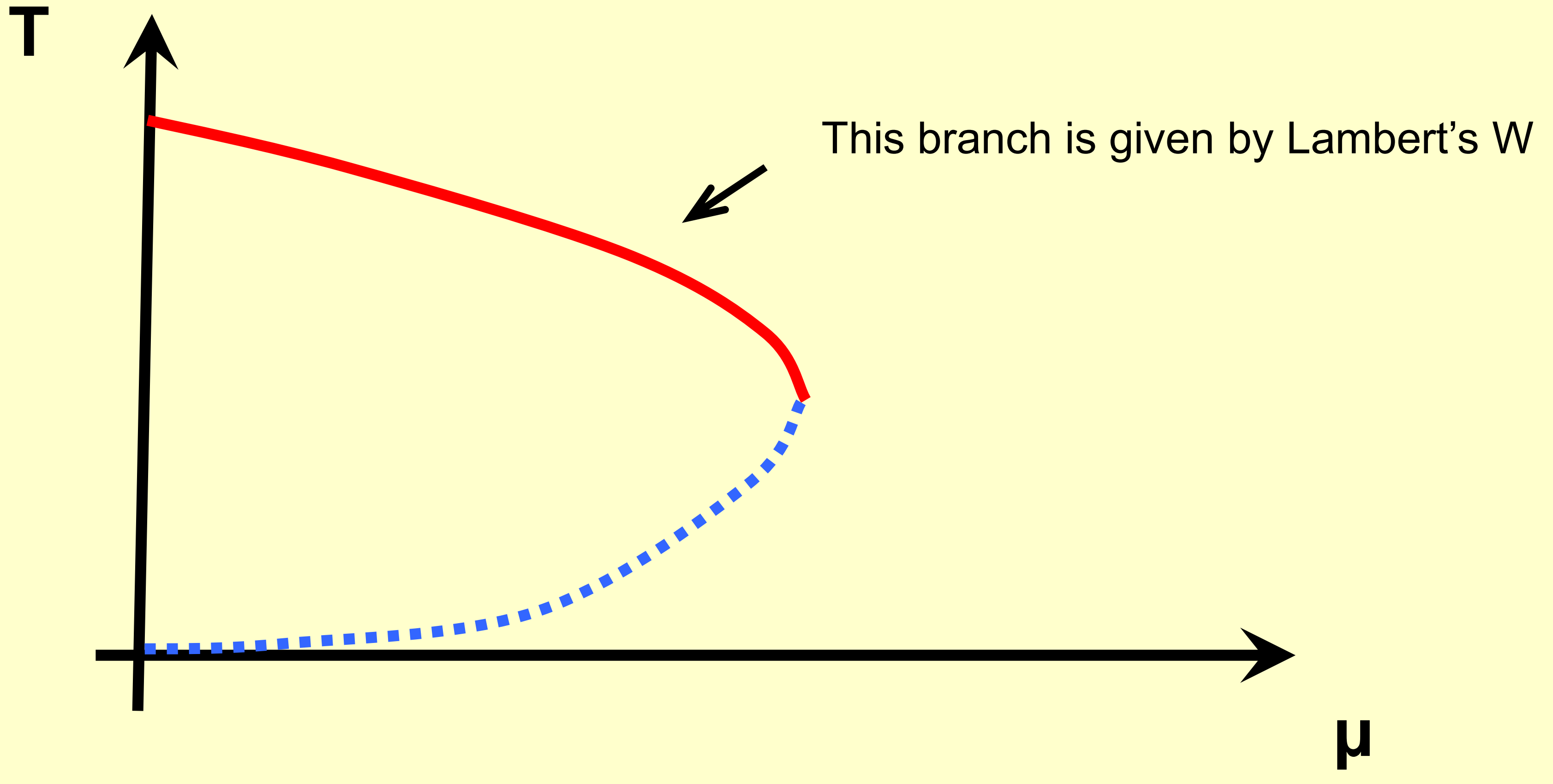
$$T_E = \left[\left(\frac{d}{\pi^2} \right)^{-\gamma} \gamma e A \right]^{\frac{1}{1+3\gamma}}$$

It is near to the zero pressure line for low chemical potential

Stringy corrections to QGP

Boltzmann approximation

Endline: last possible solution



The zero pressure line

Zero pressure in the Boltzmann approximation

$$p = cT\left(1 - \tfrac{\gamma}{1-\gamma} z\right)$$

$$\Rightarrow \; z_0 = \tfrac{1-\gamma}{\gamma} \qquad \Rightarrow \qquad \alpha_0 = \left(\tfrac{1}{\gamma} - 1\right) e^{\gamma - 1}$$

$$\mu_B = \frac{3\gamma + 1}{\gamma q} T \ln \frac{T_0}{T}, \qquad \frac{T_0}{T_E} = \left(1 - \gamma\right)^{-\frac{1}{3\gamma+1}} \; e^{-\frac{\gamma}{3\gamma+1}}$$

Endline and zero pressure line are relatively close! (T0 ≈ 1.04 TE)

E/N at the zero pressure line

General relations

$$\left.\frac{e}{n}\right|_{p=0} = \frac{\sum\left[e_{id} + \frac{1}{\gamma}\,p_{id}\right]}{\sum n_{id}}$$

$$\left.\frac{e}{n}\right|_{p=m=0} = \left(3 + \frac{1}{\gamma}\right)T\,\frac{\sum d_i \Phi_i(\tilde{\mu}_i / T)}{\sum d_i \varphi_i(\tilde{\mu}_i / T)}$$

$$\left.\frac{e}{n}\right|_{p=m=0,\ \text{Boltzmann}} = \left(3 + \frac{1}{\gamma}\right)T$$

This is independent of the value of the string tension!

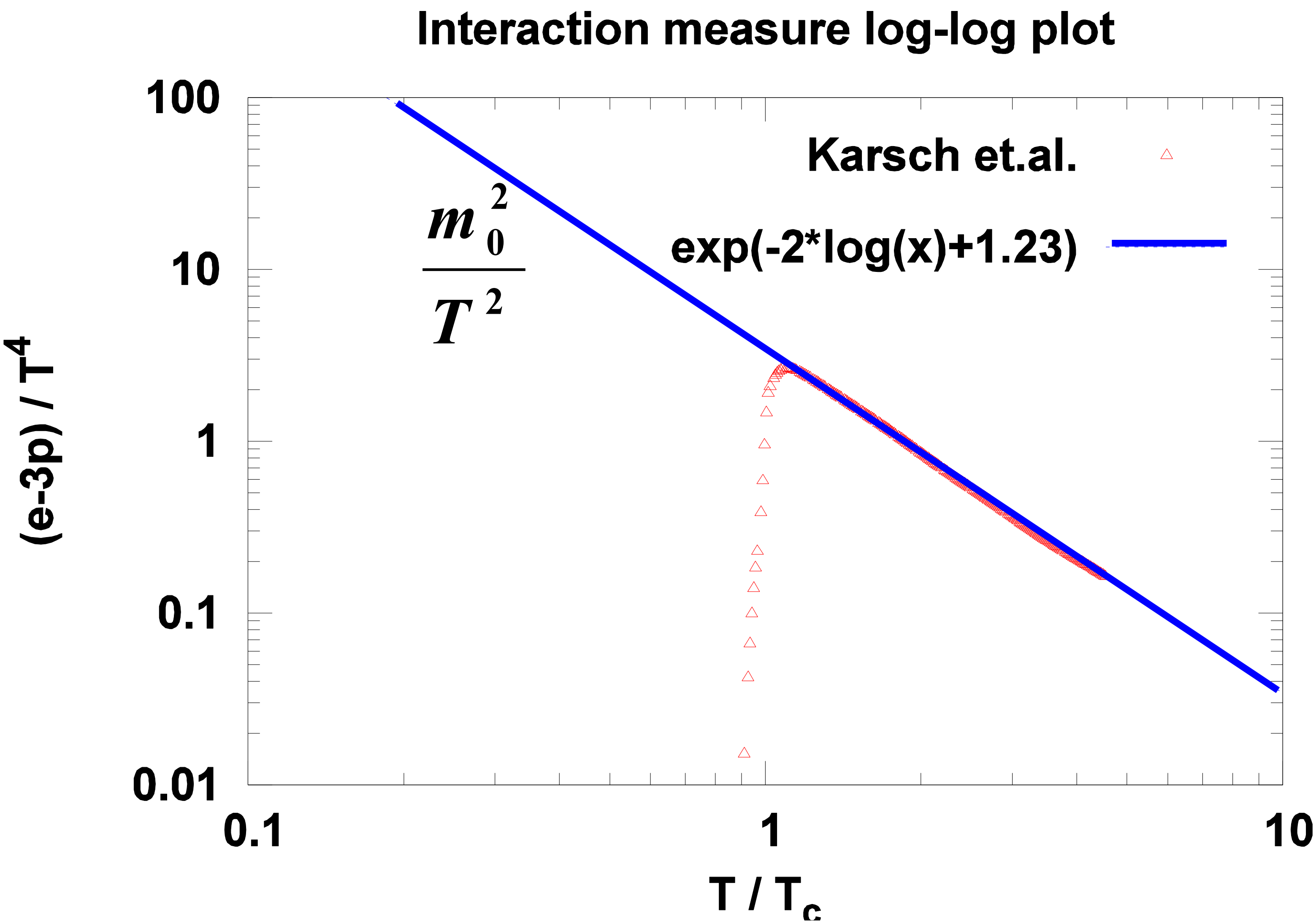
High – T equation of state

$$\frac{e-3p}{T^4} = \frac{1+3\gamma}{1-\gamma}\,\frac{A}{T^4}\,c^{1-\gamma} \sim T^{-3\gamma-1}$$

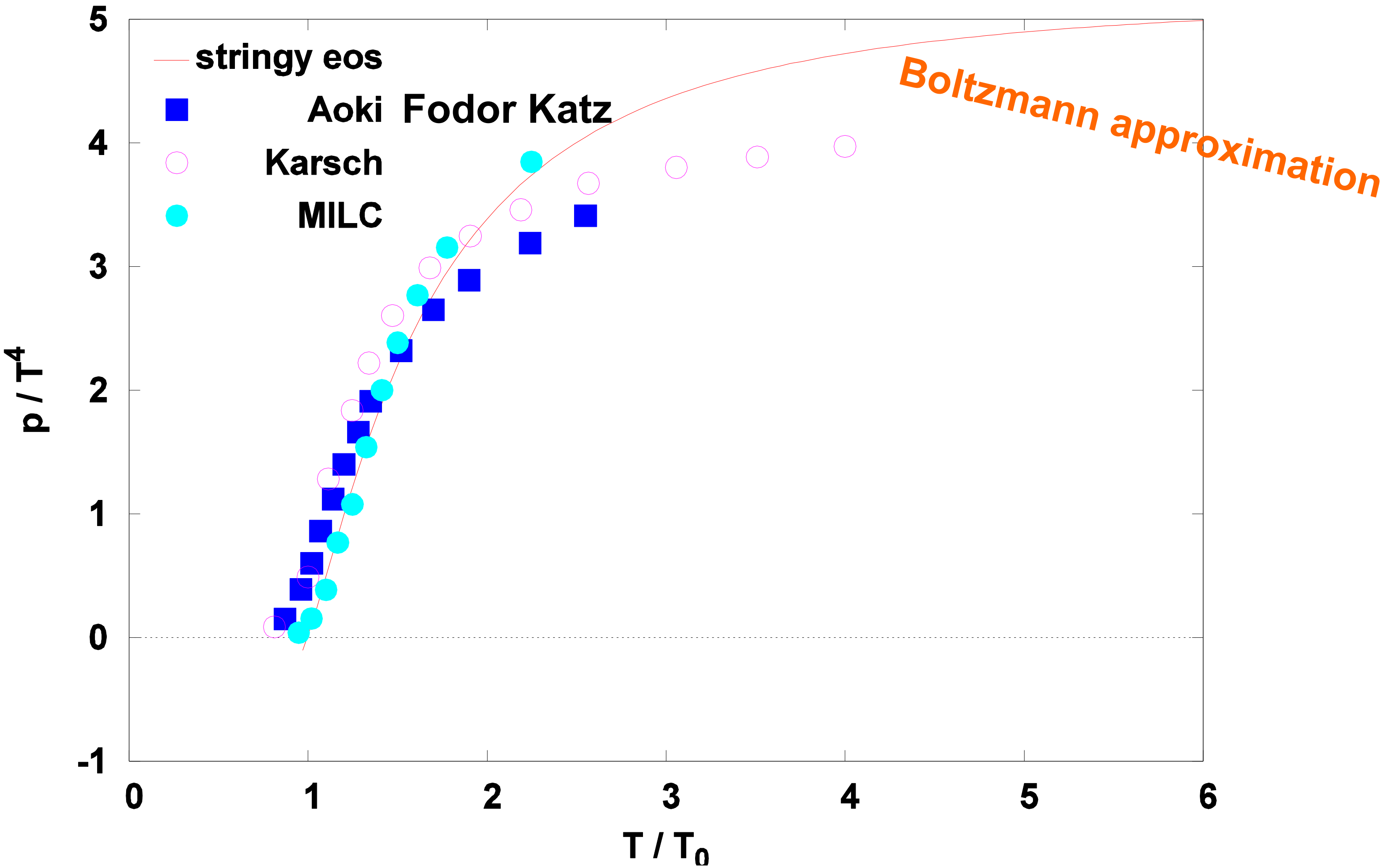
$$\text{lattice QCD}:$$

$$3\gamma + 1 = 2 \quad \rightarrow \quad \gamma = 1/3$$

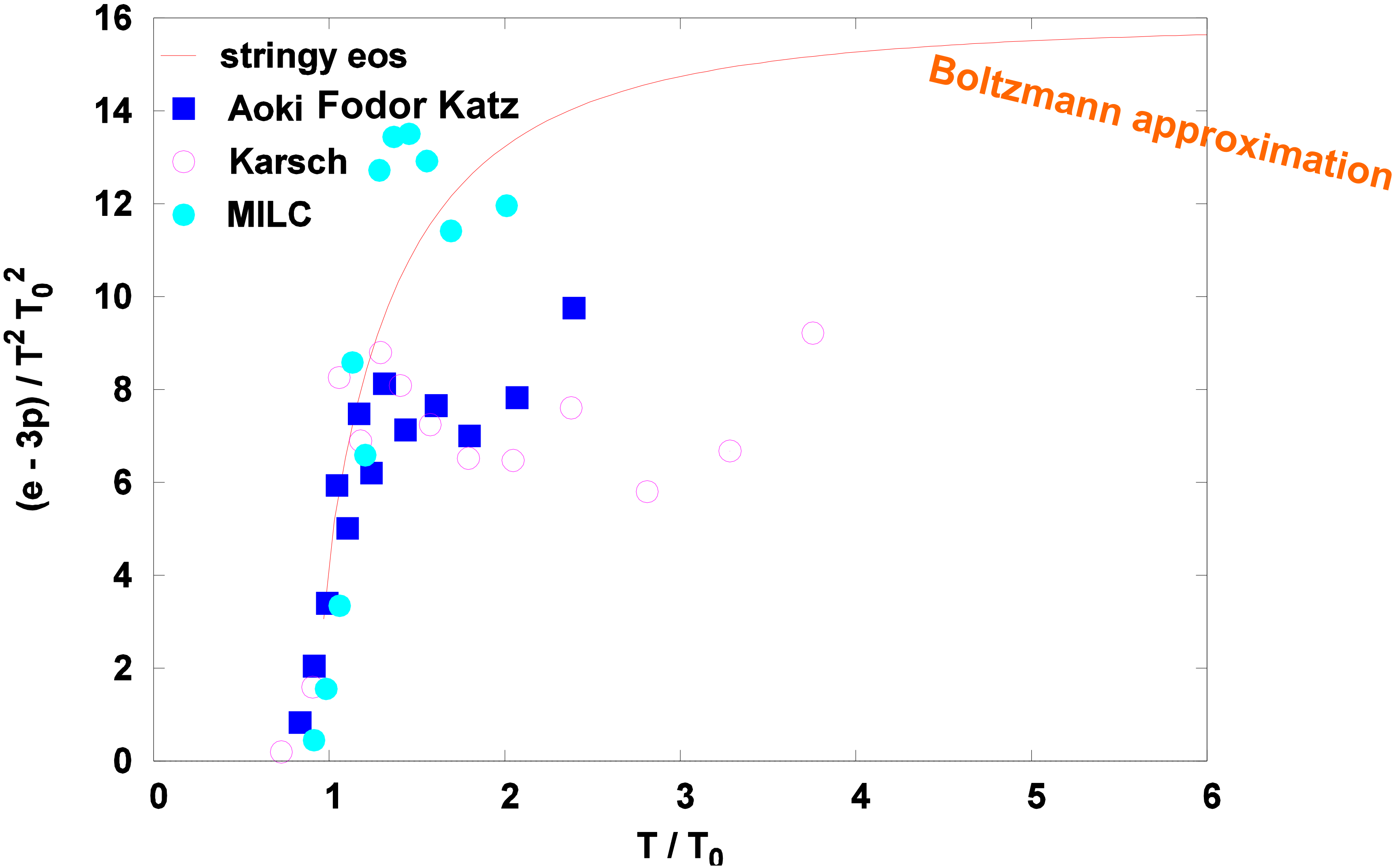
High-T behavior of lattice eos



High-T behavior of lattice eos



High-T behavior of lattice eos



Where to stop with the Boltzmann approximation?

Fermi integral ≈ Boltzmann integral for negative or zero μ.

$$q\mu_B - Ac^{-\gamma} \le 0$$

$$\frac{q\mu_B}{T} = \left(3 + \tfrac{1}{\gamma}\right)\ln\frac{T_E}{T} \le z_E = \frac{1}{\gamma}$$

$$T_E > T > T_E\, e^{-1/2} \approx 0.6\, T_E$$

The endline does not turn back!

$$T_{\max} = T_E\, e^{-1}$$

The zero pressure line

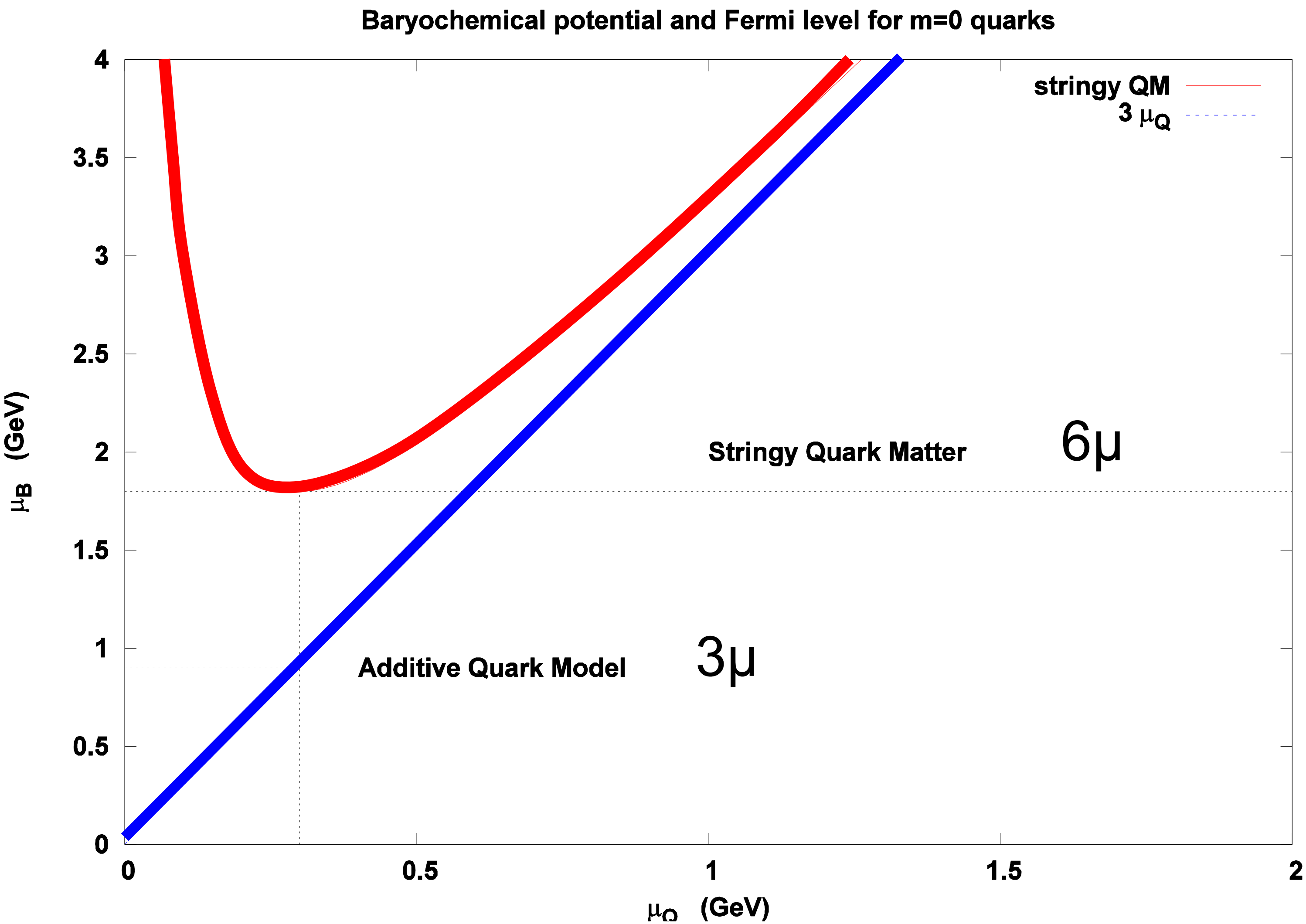
Fermions (quarks only) at T = 0

$$c = \frac{dc_q}{6\pi^2}\left(q\mu_B - Ac^{-1/3}c_q\right)^3$$

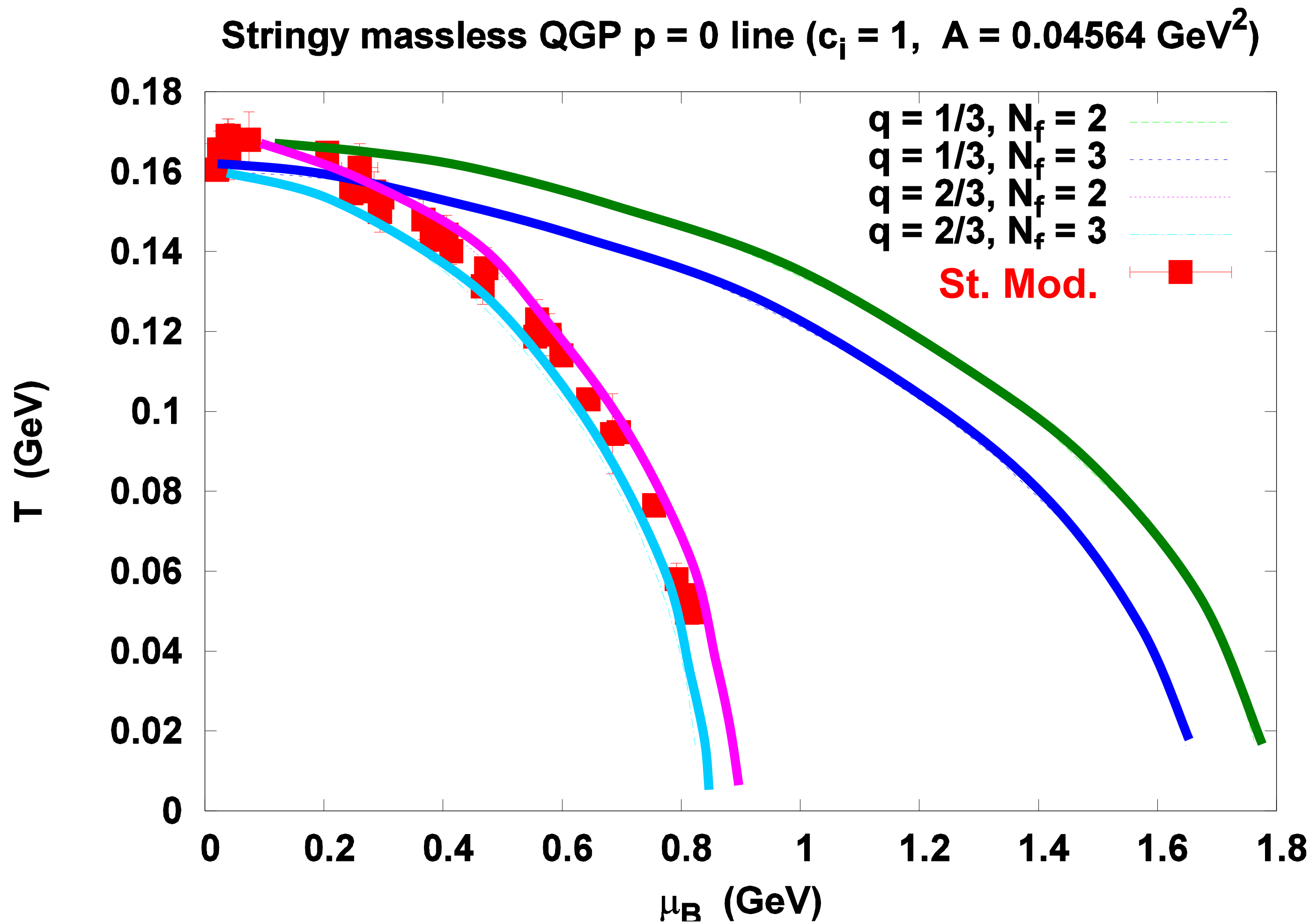
$$c = c_q\left[\frac{q\mu_B}{2B} + \sqrt{\frac{q^2\mu_B^2}{4B^2} - \frac{\widetilde{A}}{B}}\right]^3, \quad \left.\frac{e}{n}\right|_{p=0,\,T=0} = q\mu_B$$

$$\mu_0^Q = \tfrac{3}{2\sqrt{2}}\,\mu_E \approx 1.06\mu_E \approx 10.4\,T_0^{QGP} \approx 1.74\ \mathrm{GeV}$$

The endpoint at T=0



The zero pressure line




Discussion

Is acceleration a heat container?